Similarity -APPLICATION OF CIRCLE
(Golden Section)

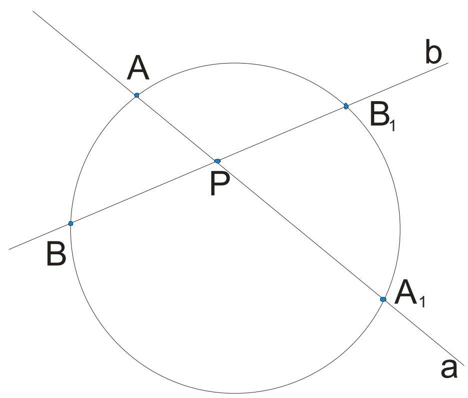
Consider the circle K and the point P in the plane of the circle. Let the lines a and b are two secant of the circle K passing through P.

It is obvious that we have three situations:

i) point P is inside circle
ii) point P is on the circle
iii) point P is outside circle

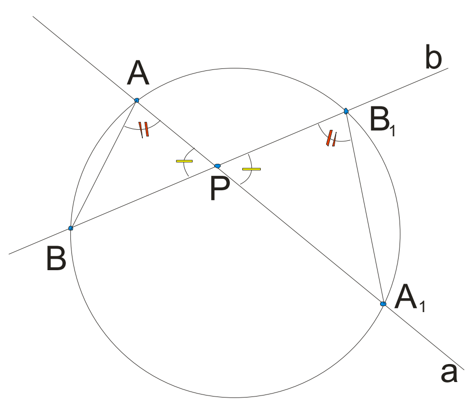
Consider the situation one by one ..

i) point *P* is inside circle



Mark with A and A_1 intersecting point of line **a** and circular line k , *and with* B and B_1 intersecting point of line **b** and circular line k.

Observe triangles ABP and A_1B_1P .

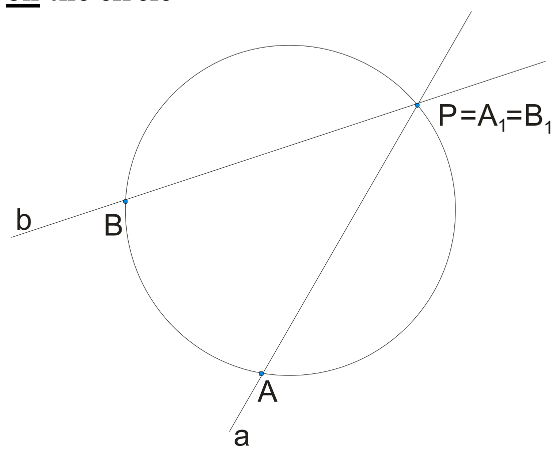


Observed triangles are **similar** because they have two equal angles: $\sphericalangle APB = \sphericalangle A_1PB_1$ (cross angle)

And $\sphericalangle PAB = \sphericalangle PB_1A_1$ the peripheral angle s of the same circular arc.

From the similarity of triangles below the corresponding proportion of sides :
$AP : BP = B_1P : A_1P \rightarrow \boxed{AP \cdot A_1P = BP \cdot B_1P}$

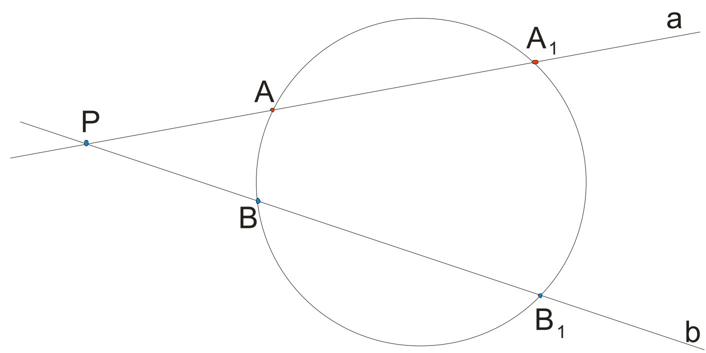
ii) point *P* is <u>on</u> the circle



Here, the points P , A_1 and B_1 match. It is obvious: $AP \cdot A_1P = BP \cdot B_1P = 0$ because $A_1P = B_1P = 0$. **So:**

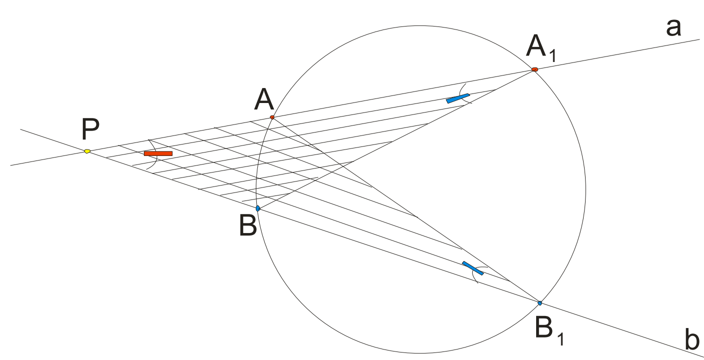
$$\boxed{AP \cdot A_1P = BP \cdot B_1P}.$$

iii) point *P* is <u>outside</u> circle



The same marks as in the first situation : A and A_1 are intersecting point of line **<u>a</u>** and circular line *k, and* B and B_1 intersecting point of line **<u>b</u>** and circular line *k.*

Observe triangles PAB_1 and PBA_1 .



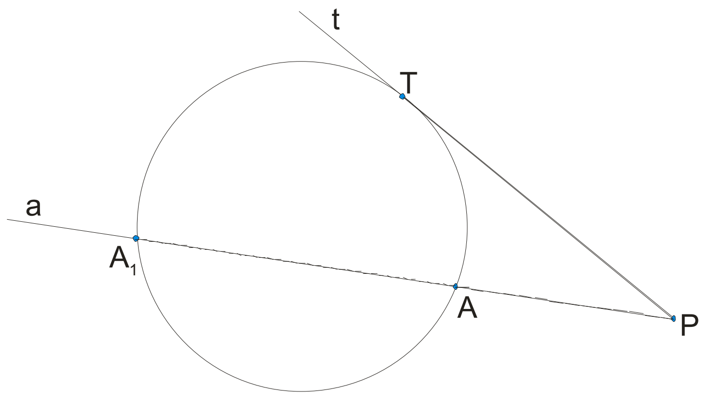
It is clear that these two triangles have a common angle at the vertex P (red in the picture) and the angles marked in blue are equal as peripheral angles of the same arc AB .

$AP : BP = B_1P : A_1P \rightarrow \boxed{AP \cdot A_1P = BP \cdot B_1P}$. And the third time we get the same conclusion:

If we have a given circle K and point P in the plane of the circle, then the product of segment that defines a circle K on any secant drawn from point P, has a constant value.

The most common label is $p^2 = PA \cdot PA_1$ --- **potency of P** in relation to circle K.

If the point P is outside the circle, it is interesting to observe the situation when we set the point P tangent to the circle and secant to the circle



This would apply: $PT \cdot PT = PA \cdot PA_1 \rightarrow \boxed{PT^2 = PA \cdot PA_1}$, that is:

Potency of P in relation to the circle K is equal to the square of the corresponding tangent.

The most interesting thing about this is the so-called **golden section.**

The **golden section** is a line segment divided according to the golden ratio: The total length ***AB*** is to the longer segment ***AC*** as ***AC*** is to the shorter segment ***CB***.

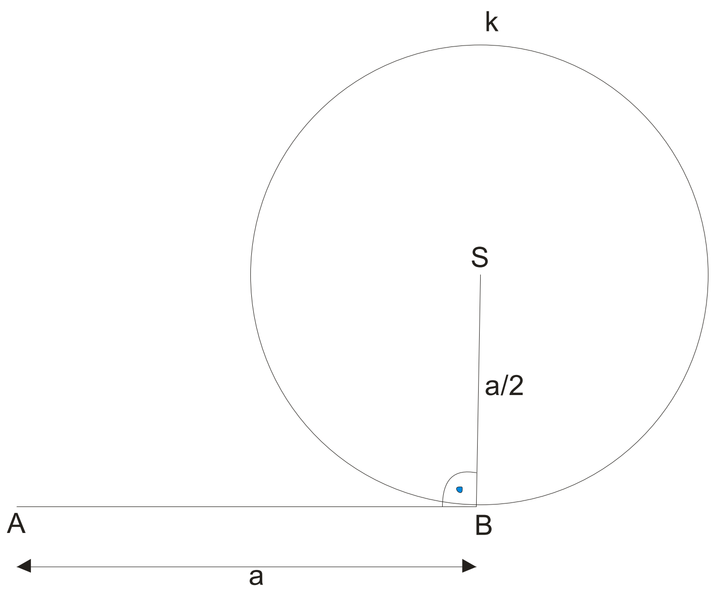
$\boxed{AC : AB = BC : AC}$ $or \;\; AC = \sqrt{AB \cdot BC}$

Of course, we will now explain how to find a structural point C along the dividing AB in the golden ratio

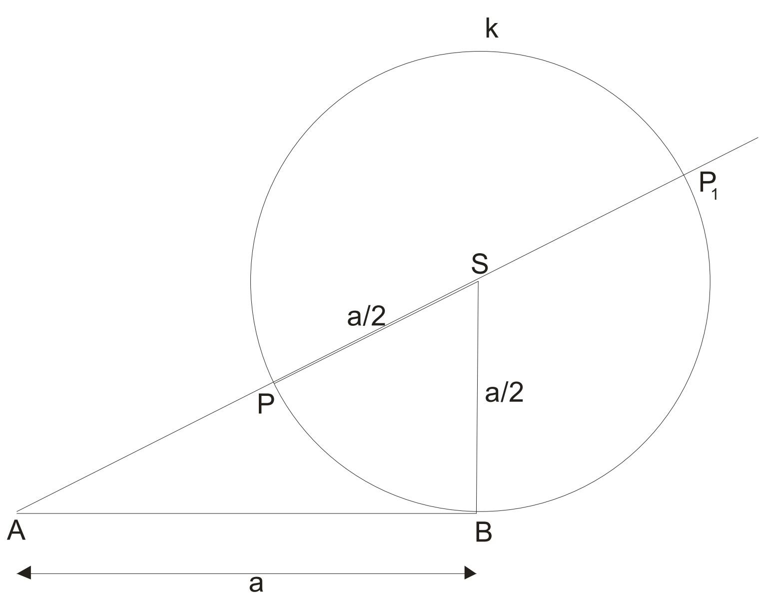
You guess that has something to do with the previous exposure…

We take an arbitrary along AB and mark $AB = a$. In point B set up the normal line and on it draw $\frac{a}{2}$. Let this be the point S.

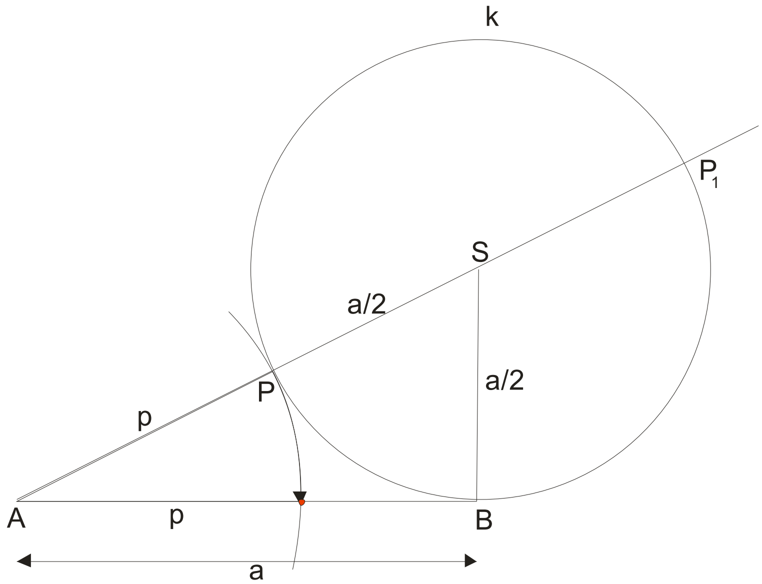
Construct a circle of radius $\frac{a}{2}$ with center in S.



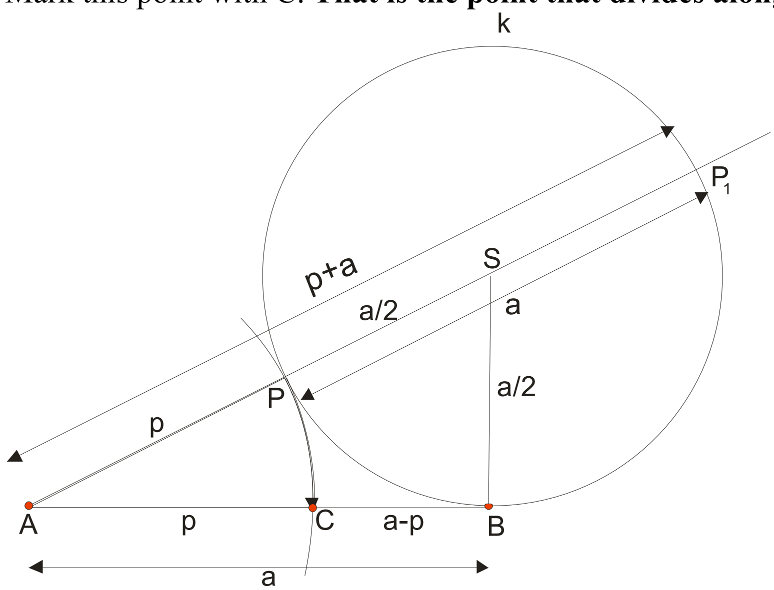
Connect S - A with line and we have secant SA . Mark the point of intersection with P and P_1.



Observe the distance between points A and P. Mark $AP = p$.



Mark this point with C. **That is the point that divides along in the golden ratio.**



The proof is simple: (watch the picture)

Based on the characteristics of potency we have $AB^2 = AP \cdot A_1P$, or $\mathrm{a}^2 = (p + \mathrm{a}) \cdot p$

From here we have :

$$\mathrm{a}^2 = (p + \mathrm{a}) \cdot p$$
$$\mathrm{a}^2 = p^2 + \mathrm{a} \cdot p$$
$$p^2 = \mathrm{a}^2 - \mathrm{a} \cdot p$$
$$p^2 = \mathrm{a}(\mathrm{a} - p) \rightarrow \boxed{p : \mathrm{a} = (\mathrm{a} - p) : p}$$

Golden Section ..

$$a^2 = (p + a) \cdot p$$

$$a^2 = p^2 + ap$$

$$a^2 - ap - p^2 = 0 \rightarrow \text{quadratic equation " by a"}, \; a = 1, b = -p, c = -p^2$$

$$a_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a_{1,2} = \frac{p \pm \sqrt{p^2 + 4p^2}}{2} = \frac{p \pm \sqrt{5p^2}}{2} = \frac{p \pm p\sqrt{5}}{2} = p\left(\frac{1 \pm \sqrt{5}}{2}\right)$$

$$a_1 = p\left(\frac{1+\sqrt{5}}{2}\right) \wedge a_2 = p\left(\frac{1-\sqrt{5}}{2}\right)$$

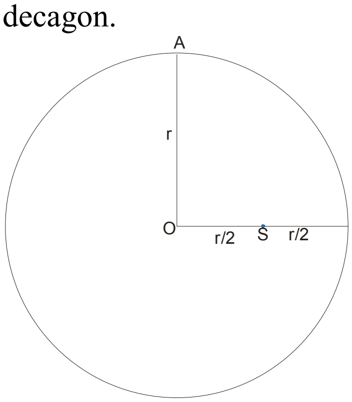
$$a_1 \approx 1{,}618033989 \cdot p, \quad \text{because} \; \frac{1+\sqrt{5}}{2} \approx 1{,}618033989$$

$$a_2 \approx -0{,}618033988 \cdot p, \; \textit{because} \; \frac{1-\sqrt{5}}{2} \approx -0{,}618033988$$

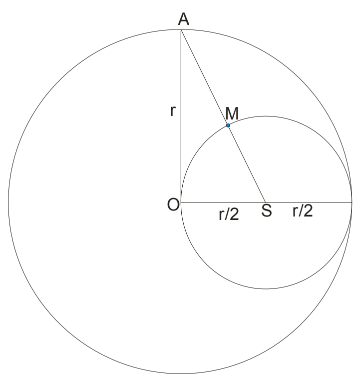
We are interested in $a \approx 1{,}618033989 \cdot p$ **, or that** $\boxed{a : p \approx 1{,}618033989}$

REMEMBER THIS NUMBER: 1,618033989

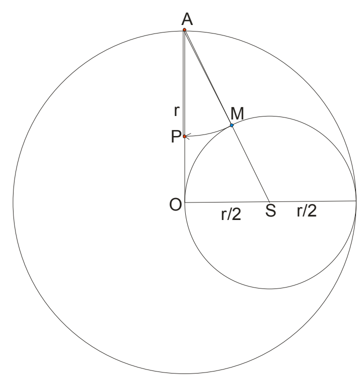
The most common task that gives teachers , related to the golden ratio, is the construction proper pentagon or decagon.



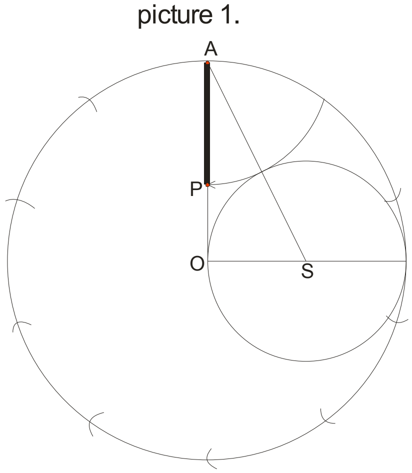
picture 1.



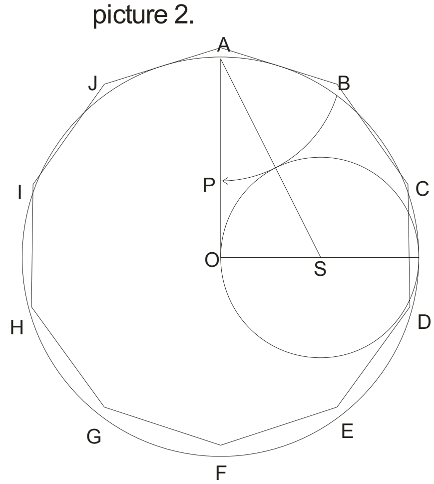
picture 2.



picture 3.

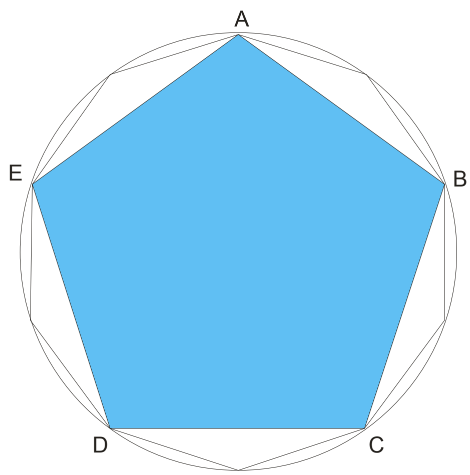


picture 4.



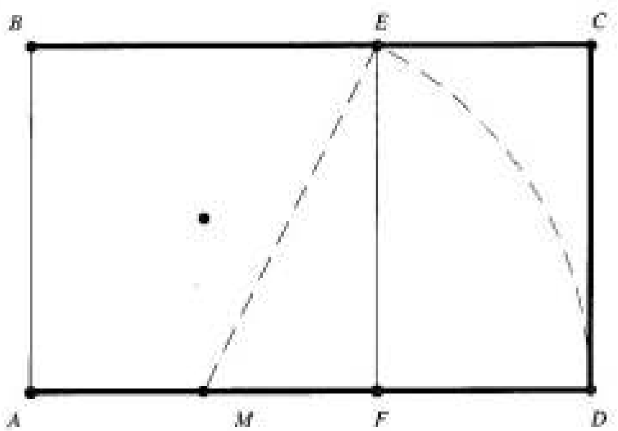
picture 5.

If the professor asks you to draw a **regular Pentagon** added to a specified radius of a circle, **you draw first Decagon and connect :**

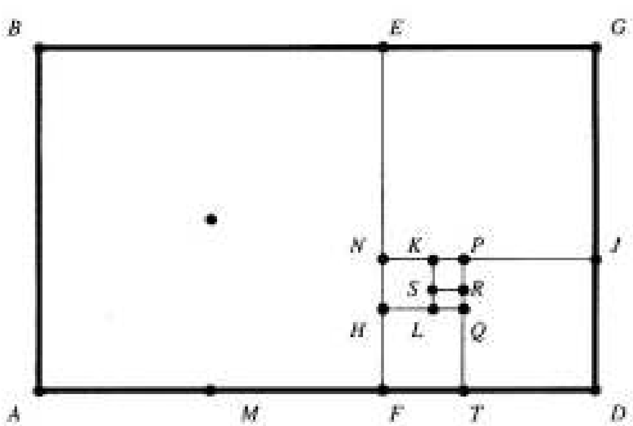


Now get back to the **golden ratio** and to tell you **some interesting facts** ...

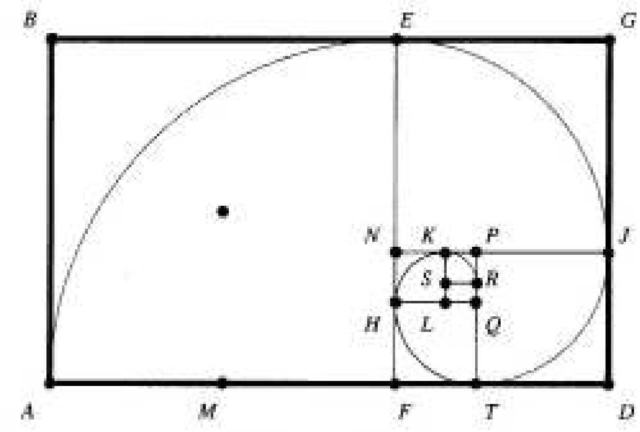
Golden Rectangle is a rectangle whose side lengths are in relation to the golden section $\boxed{a : b \approx 1,618033989}$



If we continue with the construction of the golden rectangle, we have:



Whenever I remove the square, there is a golden rectangle.



We got a so-called **golden spiral.**

The ancient architects believed that buildings have a great look if they are certain dimensions of the golden section.

It is even believed that buildings with golden section have magical powers.

The famous Parthenon in Athens was built by the golden ratio.

The Egyptian pyramids are the proportions of the golden section, the building of the United Nations ...

Golden Section is also in the works of famous musicians: Bach, Mozart sonatas, Beethoven's fifth

Schubert music ... It is in the paintings of Leonardo ...

However, the most interesting is that the golden ratio found in nature:

If you divide the number of female and male bees in the hive, we get about 1.6.
Man measure the length from the head to the navel, and to divide the length from the navel to the floor again ... 1.6.
Sunflower seeds grow in opposing spirals and the mutual relations of rotation of the diameter of 1.6.
On the box (shell) molluscs Nautilus also is the ratio of each spiral diameter of the next 1.6

*When it comes to the golden ratio, must inevitably mention the **Fibonacci sequence.***

1,1,2,3,5,8,13,21,34,55,89...

Starting from the third member, each following the sequence obtained by the two preceding paragraphs gather ..

2=1+1
3=2+1
5=3+2
8=5+3
and so on.

And not something very much apart, tell you right now ... But ... The right thing to come to light!

If we divide by two consecutive series beginning with 3 and 5, we get:

$\frac{5}{3} \approx 1,67$

$\frac{8}{5} = 1,6$

$\frac{13}{8} = 1,625$

$\frac{21}{13} \approx 1,615$

$\frac{34}{21} \approx 1,619$

$\frac{55}{34} \approx 1,617$

$\frac{89}{55} \approx 1,618$

so on...

Do you known the number $1,618033989$ **?**

The golden ratio is often denoted by the **Greek** letter **phi** **-** φ

$$\boxed{\varphi \approx 1,618033989}$$